NEURAL NETWORKS

This set of problems is intended to acquaint the student with how to find the inputs of a neural network whose outputs are known.

(1) In the two-input, two-output neural network shown in Fig. 1, the hidden neurons employ bipolar sigmoidal functions while the output neurons employ binary sigmoidal functions. If the outputs are measured as $s_1 = 0.75$ and $s_2 = 0.58$, find the inputs x_1 and x_2.

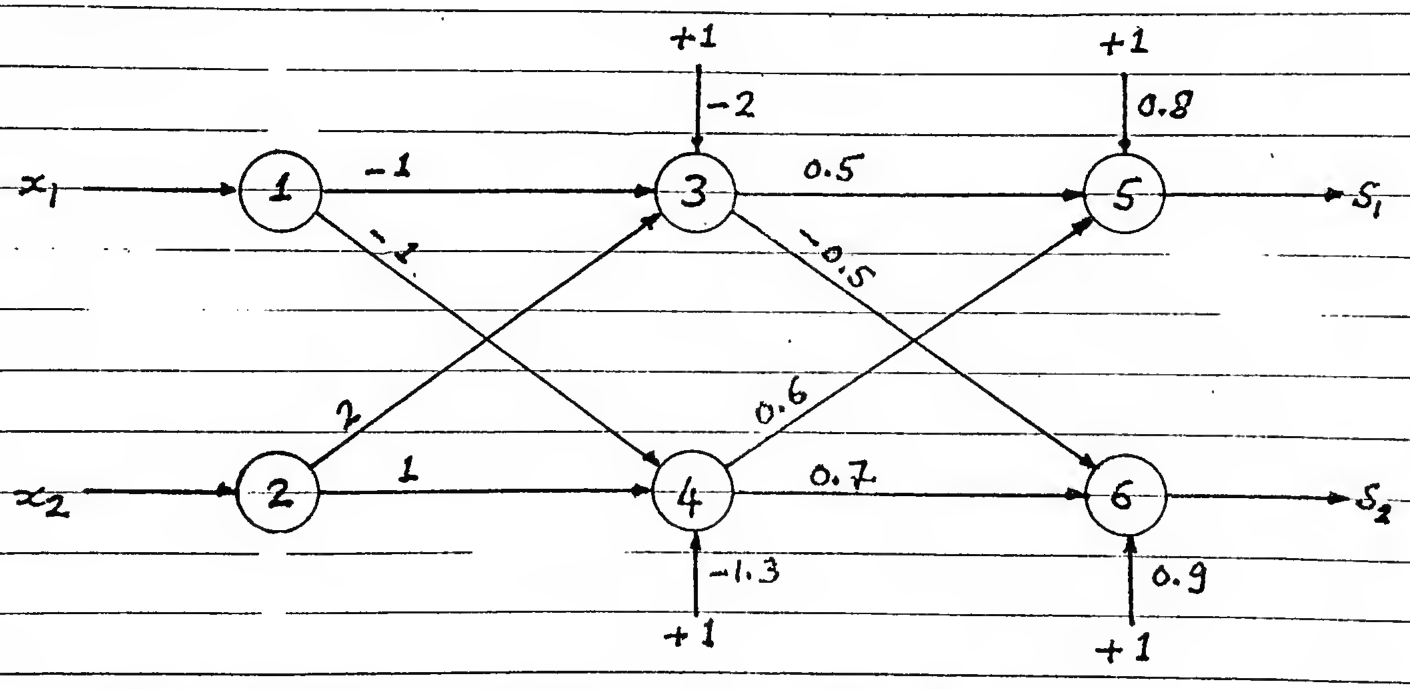

Fig. 1 Neural network for Prob. 1

(2) Consider the two-input, single-output neural network shown in Fig. 2. The hidden neurons N3 and N4 employ binary sigmoidal functions while the output neuron N5 employs a bipolar sigmoidal function. The output of N4 is twice that of N3.

and the output of N5 is $s = -0.6$. Calculate the values of the inputs x_1 and x_2.

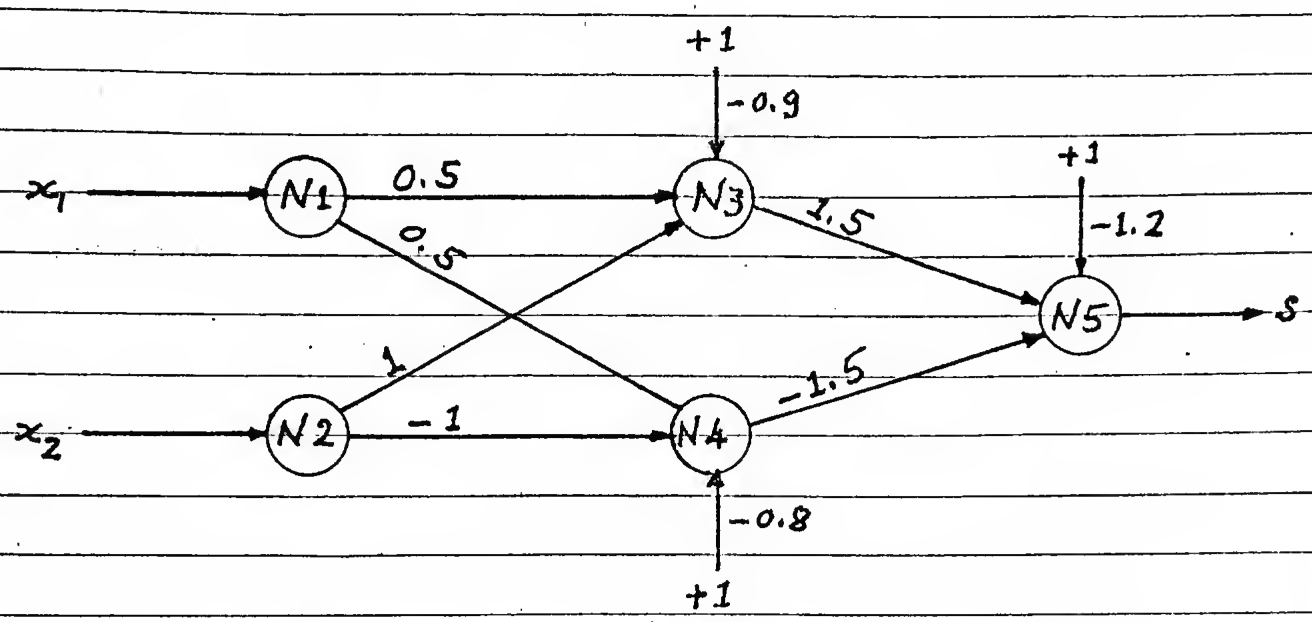

Fig. 2 Neural network for Prob. 2

(3) Consider the two-input, three-output neural network shown in Fig. 3. The hidden and output neurons employ linear functions of the form $f(x) = \alpha x$, with $\alpha = 0.2$ for each hidden neuron and $\alpha = 1$ for each output neuron. If the outputs are found to be $s_1 = 0.22$, $s_2 = -0.16$, and $s_3 = 0.115$, determine the inputs x_1 and x_2.

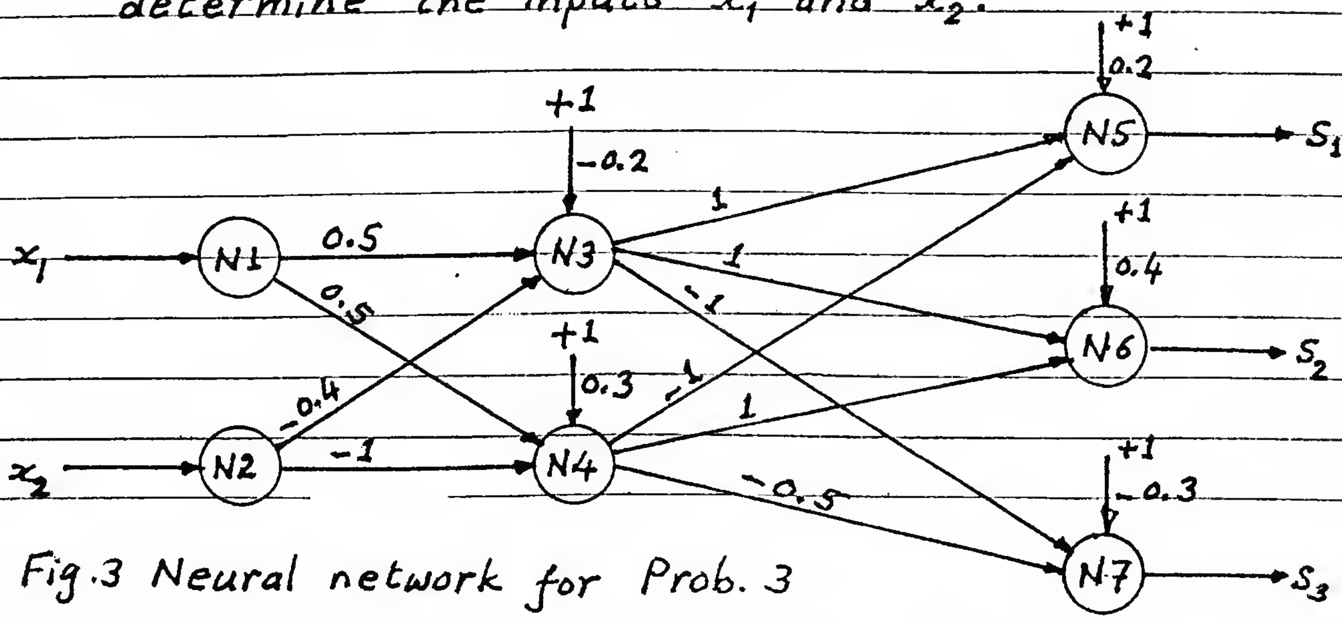

Fig. 3 Neural network for Prob. 3

Solution of Problem 1

For binary sigmoidal functions employed by output neurons 5 and 6, we obtain the activations

$$y_5 = \ln\left[\frac{s_1}{1-s_1}\right] = \ln\left[\frac{0.75}{1-0.75}\right] = 1.099$$

$$y_6 = \ln\left[\frac{s_2}{1-s_2}\right] = \ln\left[\frac{0.58}{1-0.58}\right] = 0.323$$

and we can write

$$y_5 = 0.5\,g(y_3) + 0.6\,g(y_4) + 0.8 = 1.099$$

or

$$0.5\,g(y_3) + 0.6\,g(y_4) = 0.299 \quad \ldots (1)$$

and

$$y_6 = -0.5\,g(y_3) + 0.7\,g(y_4) + 0.9 = 0.323$$

or

$$-0.5\,g(y_3) + 0.7\,g(y_4) = -0.577 \quad \ldots (2)$$

Solving Eqs. (1) and (2),

$$g(y_3) = 0.855$$
$$g(y_4) = -0.214$$

For bipolar sigmoidal functions employed by hidden neurons 3 and 4, we obtain the activations

$$y_3 = \ln\left[\frac{1+g(y_3)}{1-g(y_3)}\right] = \ln\left[\frac{1+0.855}{1-0.855}\right] = 2.549$$

$$y_4 = \ln\left[\frac{1+g(y_4)}{1-g(y_4)}\right] = \ln\left[\frac{1+(-0.214)}{1-(-0.214)}\right] = -0.435$$

and we can write

$$y_3 = -x_1 + 2x_2 - 2 = 2.549$$

or

$$-x_1 + 2x_2 = 4.549 \quad \text{...(3)}$$

and

$$y_4 = -x_1 + x_2 - 1.3 = -0.435$$

or

$$-x_1 + x_2 = 0.865 \quad \text{...(4)}$$

Solving Eqs. (3) and (4),

$$x_1 = \underline{2.819}$$

$$x_2 = \underline{3.684}$$

<u>Solution of Problem 2</u>

For $s = g(y_5) = -0.6$,

$$y_5 = \ln\left[\frac{1+g(y_5)}{1-g(y_5)}\right] = \ln\left[\frac{1-0.6}{1+0.6}\right] = -1.386$$

$$= 1.5\,f(y_3) - 1.5\,f(y_4) - 1.2$$
$$= 1.5\,f(y_3) - 1.5 \times 2 f(y_3) - 1.2$$

That is,

$$-1.5\,f(y_3) = -0.186$$

or

$$f(y_3) = 0.124$$

and

$$f(y_4) = 2 \times 0.124 = 0.248$$

For the hidden neurons,

$$y_3 = \ln\left[\frac{f(y_3)}{1-f(y_3)}\right] = \ln\left[\frac{0.124}{1-0.124}\right] = -1.955$$

$$y_4 = \ln\left[\frac{f(y_4)}{1-f(y_4)}\right] = \ln\left[\frac{0.248}{1-0.248}\right] = -1.109$$

and we can write

$$y_3 = 0.5x_1 + x_2 - 0.9 = -1.955$$

or

$$0.5x_1 + x_2 = -1.055 \quad \ldots (1)$$

and

$$y_4 = 0.5x_1 - x_2 - 0.8 = -1.109$$

or

$$0.5x_1 - x_2 = -0.309 \quad \ldots (2)$$

Solving Eqs. (1) and (2),

$$x_1 = \underline{-1.364}$$

$$x_2 = \underline{-0.373}$$

<u>Solution of Problem 3</u>

Outputs of the network,

$$s_1 = f(y_5) = y_5 = 0.22$$
$$s_2 = f(y_6) = y_6 = -0.16$$
$$s_3 = f(y_7) = y_7 = 0.115$$

Activations of the output neurons,

$$y_5 = (1)f(y_3) + (-1)f(y_4) + 0.2 = 0.22$$
$$f(y_3) - f(y_4) = 0.02 \quad \ldots (1)$$

$$y_6 = (1)f(y_3) + (1)f(y_4) + 0.4 = -0.16$$

$$f(y_3) + f(y_4) = -0.56 \qquad \dots(2)$$

$$y_7 = (-1)f(y_3) + (-0.5)f(y_4) - 0.3 = 0.115$$

$$f(y_3) + 0.5\,f(y_4) = -0.415 \qquad \dots(3)$$

Equations (1), (2), and (3), under actual operating conditions, reduce to two independent equations. Solving Eqs. (1) and (2),

$$f(y_3) = -0.27$$

$$f(y_4) = -0.29$$

Note, as expected, that the values of $f(y_3)$ and $f(y_4)$ already satisfy Eq. (3).

Activations of the hidden neurons,

$$y_3 = \frac{f(y_3)}{0.2} = \frac{-0.27}{0.2} = -1.35$$

$$= 0.5x_1 - 0.4x_2 - 0.2$$

$$0.5x_1 - 0.4x_2 = -1.15 \qquad \dots(4)$$

$$y_4 = \frac{f(y_4)}{0.2} = \frac{-0.29}{0.2} = -1.45$$

$$= 0.5x_1 - x_2 + 0.3$$

$$0.5x_1 - x_2 = -1.75 \qquad \dots(5)$$

Solving Eqs. (4) and (5),

$$x_1 = \underline{-1.5}$$

$$x_2 = \underline{1}$$